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RECALCULATION OF W AND Z RADIATIVE CORRECTIONS AND A TOP QUARK MASS ESTIMATE

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The newly published top mass prediction of 85 GeV conflicts with indirect limits on the top mass, based on precision measurements interpreted by calculations using dimensional regularization. The top quark contribution to part of the electroweak W and Z vacuum polarization tensor is recomputed using the symmetrical theory of generalised functions. The squared top mass coefficient is larger by a factor 1.44. The indirect limits on the top mass based on this computation should therefore be divided by 1.2. Further recomputation and data analysis is needed.

1. Introduction

It has been shown¹⁾ that the symmetrical theory of generalised functions^{2,3)} can be applied to computations in quantum field theory. All results are automatically finite and regularization is never needed to get rid of infinities. Integrals which are infinite in the standard sense may or may not be determinate in the generalised sense, depending on the scale transformation properties of the result. A determinacy calculus, using an indeterminate constant C , can be set up for book-keeping of the determinacy³⁾. Only determinate results can have physical meaning.

Application to QED, assuming gauge invariance and completeness of the standard model, leads to a prediction⁴⁾ of 85.1 ± 0.3 GeV for the mass of the top quark.

Indirect limits on the top quark mass have been obtained by comparing computed electroweak radiative corrections to the outcome of precision experiments⁵⁾. The results for the top mass are substantially higher than the 85.1 GeV prediction. However, the interpretation of the experiments has been based on theoretical formulæ derived by means of dimensional regularization. The symmetrical theory of generalised functions applied to the same diagrams gives results which differ by finite renormalizations.

Upon computation of the simplest term it is found that the top mass estimate is lowered. The final result to second order (one loop) is a reduction of the top mass estimate by a factor of 0.84. This removes at least part of the conflict between the 85.1 GeV prediction and the indirect experimental data, as commonly interpreted.

On basis of dimensional regularization it is not possible to predict the top mass. It is therefore inconsistent to confront the top mass prediction⁴⁾ with experimental interpretations depending essentially on dimensional regularization.

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It is not sufficiently appreciated that all regularization methods are arbitrary by finite renormalizations. These can be fixed by convention, but there is no reason to suppose one convention to be superior to others. Results depending on arbitrary conventions cannot serve as basis for physical predictions. The reasons for preferring a form of dimensional regularization are irrelevant when gauge invariance has been secured by other means¹⁾. Observability of a physical quantity does not imply that its calculation is independent of the regularization method.

The symmetrical theory of generalised functions, applied to computations in quantum field theory, is not arbitrary by finite renormalizations. The arbitrariness of the product definition is resolved by imposing simplicity arguments on the underlying mathematical theory, and the values of integrals follow uniquely from the product definition.

Our preliminary conclusion is that the experimental data, interpreted on basis of dimensional regularization, cannot be used to reject the 85.1 GeV top mass prediction. It is not possible at present to be more explicit, since it is not clear from the literature how the data reduction has been done and in how far the results depend on the use of theoretical formulæ in which the use of dimensional regularization does make a difference.

Other calculations and some of the data reduction should be redone as well before a firm conclusion can be reached. This entails much work, so the top quark (if it is near 85 GeV) might be found before this can be accomplished.

2. W and Z vacuum polarization by fermions

In this letter we compute the fermionic contributions to the vacuum polarization, in particular the correction to the parameter ρ defined to zeroth order by^{6,7)}

$$\rho := \frac{m_W^2}{\cos^2 \theta_w m_Z^2}. \quad (1)$$

In general the vacuum polarization tensor $\Pi^{\mu\nu}(k)$ can be written as

$$\Pi^{\mu\nu}(k) = A(k^2)g^{\mu\nu} + B(k^2)k^\mu k^\nu. \quad (2)$$

The correction to ρ is then found to be⁶⁷⁾

$$\Delta\rho = \frac{A_{ZZ}(0)}{m_Z^2} - \frac{A_{WW}(0)}{m_W^2}, \quad (3)$$

where the subscripts indicate the gauge boson type.

The fermionic contribution to the vacuum polarization is found from the fundamental two fermion-boson vertex and the corresponding loop diagram

$$\text{Feynman diagram to be added} \quad (4)$$

which is needed for this letter only at boson momentum $k = 0$. Here g , c_v , and c_a are coupling constants. Substitution of the Feynman rules for Dirac fermions gives

$$\Pi_{\mu\nu}(0) = -g^2 \text{Tr} \int \frac{d^4 p}{(2\pi)^4} (c_v \gamma_\mu - c_a \gamma_\mu \gamma_5) \frac{\not{p} + m_1}{p^2 - m_1^2} (c_v \gamma_\nu - c_a \gamma_\nu \gamma_5) \frac{\not{p} + m_2}{p^2 - m_2^2}. \quad (5)$$

After evaluating the trace it is convenient to contract with $g^{\mu\nu}$ to obtain

$$4iA(0) = \Pi_\mu{}^\mu(0) = 2g^2 \int d^4 p \frac{(c_a^2 + c_v^2)p^2 + 2(c_a^2 - c_v^2)m_1 m_2}{(p^2 - m_1^2)(p^2 - m_2^2)}. \quad (6)$$

For W bosons the dominant contribution is obtained by taking the fermions to be a top and a bottom quark. To sufficient accuracy the bottom quark is massless. The Cabibbo-Kobayashi-Maskawa matrix element then equals one. (Considering all massless quark flavours the CKM matrix elements sum to one by unitarity, giving the same result). Combining the denominators by the standard Feynman trick, and substituting the values of the integrals from the appendix, yields after evaluation of the trivial auxiliary integration

$$A_{\text{WW}}(0) = \frac{3g_w^2 m_t^2}{32\pi^2} (1 - \log m_t^2 - C), \quad (7)$$

with an additional factor 3 to account for the colours of the quarks. It is seen from the appearance of a C that the result is indeterminate.

For the Z boson the dominant correction comes from the top-antitop loop, which can be evaluated directly. The result is

$$A_{\text{ZZ}}(0) = \frac{3g_w^2 m_t^2}{32\pi^2 \cos^2 \theta_w} \left(\frac{1}{2} - \frac{4}{3} \sin^2 \theta_w + \frac{16}{9} \sin^4 \theta_w - \log m_t^2 - C \right). \quad (8)$$

In contrast to the corresponding determinate photon mass correction²⁾ which has only c_v vector coupling, the axial c_a contribution is indeterminate. The corrections for the members of the W triplet are not independent. Substituting (7) and (8) into (3), the correction produced by the top quark to the ratio ρ defined in (1) is seen to be determinate

$$\Delta\rho_t = \frac{3\sqrt{2}}{16\pi^2} G_F m_t^2 \left(1 + \frac{8}{3} \sin^2 \theta_w - \frac{32}{9} \sin^4 \theta_w \right). \quad (9)$$

This is no surprise since the corresponding computation^{6,7)} using dimensional regularization yields a finite answer which must agree with (9) up to a finite renormalization¹⁾. The result has been expressed in terms of the Fermi coupling constant $G_F = g_w^2/4\sqrt{2} m_W^2$

Comparing the result using symmetrical generalised functions to the result of the same calculation using dimensional regularization^{6,7)}

$$\Delta\rho_t = \frac{3\sqrt{2}}{16\pi^2} G_F m_t^2, \quad (10)$$

one sees that (using $\sin^2 \theta_w = 0.2325^8$) the coefficient of the squared top mass increases by a factor of 1.44, so all else remaining the same the top mass limits predicted on basis of (10) must be divided by 1.2. Given the large uncertainties inherent in the experimental data this removes part (and perhaps all) of the discrepancy of the 85.1 GeV prediction and indirect estimates ranging from 100-150 GeV. Conversely, if the top mass prediction from⁴⁾

$$m_t^2 = 9m_W^2/8, \quad (11)$$

is substituted the prediction

$$\Delta\rho_t = \frac{27g_w^2}{512\pi^2} \left(1 + \frac{8}{3} \sin^2 \theta_w - \frac{32}{9} \sin^4 \theta_w\right) \approx 0.0023 \cdot 1.44 = 0.0033, \quad (12)$$

is obtained. This cannot yet be compared with experiment since the other contributions to $\Delta\rho$ have to be recomputed as well. It is unclear at present how this affects the interpretation of experiments. (Precision measurements of $\sin^2 \theta_w$ often give several results for different top mass assumptions, but the coefficient multiplying m_t^2 may have to be recomputed as well). This will affect all top mass coefficients from quadratically divergent diagrams.

3 Conclusion

It is inconsistent to compare the top quark mass prediction⁴⁾ with indirect limits on the top mass based on computations using dimensional regularization. The results obtained by computing with generalised functions differ by finite renormalizations from those obtained by means of dimensional regularization.

Recomputation of the W and Z vacuum polarization tensor to one loop removes at least part of the conflict between the 85.1 GeV top mass prediction and experimental data interpreted on basis of dimensional regularization. Further recomputation and data analysis is needed to obtain definitive results.

Appendix The imaginary parts of the integrals one needs were derived in ref 2. The results are (correcting for the modified sign of $g_{\mu\nu}$)

$$\int d^4p \frac{1}{(p^2 - a^2)^2} = -i\pi^2(\log a^2 + C), \quad A1$$

$$\int d^4p \frac{p^2}{(p^2 - a^2)^2} = -2i\pi^2 a^2(\log a^2 + C - \frac{1}{2}) \quad A2$$

with C the indeterminate constant and the $i\epsilon$ in the denominator understood. Purists may read $\log a^2$ as $\log a^2/M^2$, with M an arbitrary unit of mass, but by definition³⁾ this does not influence any physical result.

It is conventional to normalize the coupling constants as

boson	g^2	c_v	c_a
W	$g_w^2/8$	1	-1
Z	$g_w^2/4 \cos^2 \theta_w$	$1/2 - 4/3 \sin^2 \theta_w$	$1/2$

The CKM element in the W-coupling $U_{tb} = 1$ for our computation, as explained above.

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Note added 18-05-94:

If recent top mass claims near 170 GeV are confirmed, the mass sum rule derived in ref.4 indicates the need for adding charged (Higgs?) bosons to the standard model, with a mass given by

$$m_H^2 = \frac{8}{3}m_t^2 - 3m_W^2$$

which yields $m_H = 205$ GeV for 170 GeV top quarks. (assuming again that no additional charged fermions and bosons exist). The possible observation of heavy top quarks does not affect the conclusions of this letter, but it will affect predicted Higgs boson masses.

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